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THE MULTIYEAR CAPITAL BUDGETING PROBLEM: A LAGRANGIAN RELAXATIO--ETC(U)
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(6) THE MUTIYEAR CAPITAL BUDGETING PROBLEM: A LAGRANGIAN RELAXATION APPROACH

by

(10) R.D. Armstrong
W.D. Cook*
D. Kung*

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*York University, Ontario, Canada

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A. Charnes, Director
Business-Economics Building, 203E
The University of Texas at Austin
Austin, Texas 78712
(512) 471-1821

ABSTRACT

This paper presents a new approach to the classical multiyear capital budgeting problem. Three basic operational principles, arising out of an actual implementation of such a model, are specifically addressed in this approach. These principles are: (1) All budgets except that of the first year possess a degree of flexability; (2) the user favours a series of near optimal solutions to a single optimal solution; and (3) an effective low cost procedure for dealing with large problems is required. Using the particular example of capital budgeting in the area of highway maintenance, an algorithm is developed and computational results for a variety of problem sizes are presented.

Key Words: Capital Budgeting, Multiple Choice Knapsacks, Langrean Relaxation.

1. INTRODUCTION

Multiyear capital budgeting (CB) models for project selection are common in the MS literature, and have been applied in a variety of different areas. These models, generally expressed as zero-one integer programming problems, have been solved using a plethora of different solution procedures, and have met with varying degrees of success, which for the most part has been dependent on the particular application involved. Unfortunately, failure has prevailed more often than success. The current research was stimulated by recent attempts on the part of the authors to apply a CB model to a particular problem. During the implementation phase, conducted under close scrutiny of the client, certain operating realities materialized. The purpose of this paper is to highlight these realities, and to present procedures for dealing with them in a pragmatic way.

The first of these realities or principles is that the user often prefers several good solutions to his problem as distinct from a single "optimal" solution. Considerations beyond the scope of the model often dictate that certain solutions are operationally feasible while others are not. Second, budget figures beyond that of the first year of the planning horizon are generally not known in advance, but rather must be estimated. Consequently, the strict requirement to not exceed these figures can be relaxed. Third, with very large capital budgeting problems it is often the case that either sophisticated optimization packages are not available or that those which are available cannot solve problems of such magnitudes effectively and at a low cost.

In section 2 we describe a particular capital budgeting problem investigated by the authors. Section 3 presents a multiyear capital budgeting formulation for dealing with this problem. Section 4 elaborates on the three operating principles described above, and provides a reformulated version of the model which is appropriate for dealing with these principles An algorithm tailored to these principles is then described. Section 5 discusses the implementation process and computational results.

2. CAPITAL BUDGETING IN PAVEMENT MAINTENANCE

A problem of significant interest in recent years is that involving the effective allocation of maintenance budgets in the area of pavement rehabilitation. With millions of dollars at stake annually, a tremendous responsibility has been delegated to transport agencies to initiate cost-effective maintenance strategies. The desire on the part of these agencies to develop better planning tools has been stimulated by two phenomena. These phenomena, which have materialized primarily during the past five-ten years, are described below.

The first of these phenomena is that the means for evaluating different maintenance alternatives is becoming more and more prevalent.

This has resulted from several incentives. Specifically, numerous studies have been conducted in different countries pertaining to the performance of pavements in terms of such parameters as age, thickness, subgrade structure and traffic. See for example, [3], [7], [9]. Additionally, many highway departments have created mechanisms for collecting data relating to the structural adequacy of pavements within their jurisdiction. This data has been used to construct functions for predicting performance associated with various rehabilitative strategies.

The second of these phenomena is one of need. The desire for closer monitoring and more effective planning in the area of pavement maintenance is much greater now than was the case a few years ago. The reason is simply one of resources. The proportion of treasury dollars being allocated toward highway construction and maintenance is substantially less at present than it was in the past.

There exist very few sophisticated models for capital budgeting in this environment. In the most basic of situations, a particular department will have standard strategies which it has been in the practice of using on particular road categories in the past. These strategies will be applied to those roads declared to be the worst (or expected to be so over the ensuing 5 to 10 year period) up to the point where the budget is absorbed. This approach is optimal under the "grandfather" criterion. Moving towards a more sophisticated approach, the department may have fairly accurate data on pavement performance, allowing one to assess the impact of various alternatives. Various ranking and benefit/cost ratio approaches have been attempted using these data. Here again, however, the model tends to operate outside of the budget framework to a great extent.

Priority planning in pavement maintenance can be viewed as a two phase process. The first phase involves the utilization of available data on pavement condition ratings to construct performance functions. These functions are used to forecast the structural rating at various points in time during the planning horizon. In the second phase of planning these performance measures are used to determine, from among the many possible rehabilitative options available, that option for each proposed project which maximizes the aggregate performance of the highway system. The CB model examined herein concentrates on this second phase. Details of this follow in sections 3 and 4. The remainder of the current section gives a brief overview of the phase 1 process in the particular application under discussion.

Available data on pavement surface condition normally relates to both structural and geometric adequacy. Structural adequacy is a function of cracking, roughness, frost heaving, etc. Geometrics pertains to lane and shoulder width, slope and crossfall corrections. In terms of pavement maintenance we concentrate on structural adequacy only, since this was the area of primary concern in the system investigated.

Various indices/parameters are used to characterize structural adequacy. The condition rating (PCR) used in this study was a measure which accounted for all forms of structural distress. In the system studied by the author, data in the form of (PCR, pavement age) pairs was available corresponding to different traffic categories, different geographical regions, and pavement thicknesses.

This data was used to construct a family of performance functions of the form

$$P_o - P = KA^B. \tag{2.1}$$

In (2.1) P_o is the rating when the pavement was new, P is its rating after A years, and (K,B) are regression coefficients. This family of curves is similar to those used in the ASSHTO road test [9]. Further details on these functional forms can be found in [1] and [2] which dealt with single year planning problems.

The planning horizon for highway maintenance strategy selection normally covers either a 5 or 10 year period. Given a set of estimated budgets, planners must create a maintenance program which stays within these limits and which will provide the highest possible level of service from the highway system. Generally, this amounts to selecting a rehabilitative strategy (a maintenance alternative together with an action year or years in which to implement that strategy) for each highway section in the system.

For each strategy a benefit, R, is calculated based on the area under the PCR curve plus a salvage value. The set of possible strategies is further reduced by imposing a minimum acceptance level on the rate of benefit. Out of the remaining strategies, a final set of allowable strategies is then selected depending on the ratings in the benefit/cost ratio. Commonly, one would restrict the allowable number of strategies to be between 50 and 200. This means that for every road section, there are usually 50 to 200 strategies treated as input to the model. Exceptions are usually initiated by the planners and can easily be incorporated into the system.

In this paper we propose a model for assigning strategies to highway projects or sections which maximizes the overall performance of the system while obeying budget restrictions. The model given is capable of handling large problems, is relatively inexpensive to operate in terms of computer running and storage costs, and can be modified to allow for certain regional or sectoral requirements. This model is described in the following sections.

3. A MULTIPLE CHOICE KNAPSACK MODEL

Let the road sections to be considered for rehabilitation during the 10 year horizon be numbered 1, 2, ..., K. Let the possible rehabilitative strategies for section k be numbered 1, 2, ..., N(k). R_{kj} is the total return or benefit from using strategy j on road section k, as described previously. B^y, y = 1, ..., 10 are available budgets. The cost of implementing strategy j on road section k in year y is denoted by C^y_{kj}. For years where no major rehabilitative work is scheduled, C^y_{kj} will represent routine maintenance only. Let

$$X_{kj} = \begin{cases} 1 \text{ if strategy } j \text{ is used on road section } k \\ 0 \text{ otherwise.} \end{cases} \tag{3.1}$$

For completeness, a "do-nothing" strategy is added to each class of strategies for each road section.* For the "do-nothing" strategy, the cost for each year will be the maintenance cost incurred. This allows the imposition of the multiple choice constraints,

$$\sum_{j\varepsilon N(k)} X_{kj} = 1, \text{ for each class } k. \tag{3.2}$$

A model for the problem discussed in the previous section can then be defined as follows:

$$Z = \text{maximize} \sum_{k=1}^{K} \sum_{j\varepsilon N(k)} R_{kj} X_{kj}$$

$$\text{subject to} \quad \sum_{k=1}^{K} \sum_{j\varepsilon N(k)} C^y_{kj} X_{kj} \le B^y, \; y = 1, \ldots, 10$$

$$\sum_{j\varepsilon N(k)} X_{kj} = 1, \; k = 1, \ldots, K \tag{3.3}$$

$$X_{kj} = 0, 1, \text{ for all } j\varepsilon N(k), \; k = 1, \ldots, K.$$

The index sets $N(k)$ defining the multiple choice classes are mutually exclusive. $R_{kj} \ge 0$, $C^y_{kj} \ge 0$, $B^y \ge 0$, for all $j\varepsilon N(k)$, $k = 1, \ldots, K$, and $y = 1, \ldots, 10$.

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*Except in the case of those sections where a fixed strategy has been locked in, hence the road is already on a program.

4. ALGORITHMIC PROCEDURE

In this section, a detailed discussion of certain particular requirements of the problem involved will first be presented. Then, a specialized algorithm for the problem, taking the indicated requirements into consideration, will be discussed.

Operational Considerations

During the initial analysis of the problem, it was observed that, other than the budget for the first year, yearly budgets are all projected values. These values will actually be used as proposals for the years to come. It is apparent then that there is no need for these nine yearly resource constraints to be satisfied precisely. That is, it is acceptable for annual expenditures to exceed estimated budgets by a reasonable amount for each of years 2, ..., 10. This additional amount can usually be expressed as a percentage of the yearly figure. At all times, however, the budget constraint for the first year must be satisfied completely since this level of funding has already been fixed.

A second consideration to be made in this problem was that the user preferred to see a variety of good solutions as distinct from a single optimal value. The reason for this is simply that there are certain practical aspects to the problem which the model could not conveniently capture. For example, hot mix patching is commonly done on a road section where another section nearby is being resurfaced. Patching an isolated section a hundred miles from available asphalt sources and gravel pits would never be done. Consequently, the user would prefer a program that contained as few irregularities such as this as possible. Other user imputed considerations are possible as well with the multiple solution facility.

In addition to the above, there is also a minimum acceptable performance requirement in terms of the same unit as the benefit, R_{kj}, for the overall model. This becomes the lower bound on the objective value of the model. In other words, any potential solution has to achieve an objective value exceeding that of the minimum in order to be considered as a feasible solution in the practical sense. As a matter of fact, any solution to the above model which satisfied the first constraint and has a objective value higher than the minimum can be treated as a candidate solution for the problem, as long as the yearly expenditures for the last nine years are considered as acceptable. The decision-maker will then have to select the best solution available, taking into considerations factors that cannot be captured by the mathematical model itself.

The Algorithm

After taking the requirements discussed above into consideration, the authors believe that the most suitable solution technique for this problem is a Lagrangian relaxation approach. It deviates from conventional Lagrangian relaxation approaches in the sense that only nine of the ten budget constraints are incorporated into the objective function. Also, the integrality restriction is retained. Then, the

Lagrangian dual problem of Z can be defined as follows:

$$Z_D = \underset{U}{\text{minimize}}\ \underset{X}{\text{maximize}} \sum_{k=1}^{K} \sum_{j \in N(k)} R_{kj} X_{kj} - \sum_{y=2}^{10} U^y \left(\sum_{k=1}^{K} \sum_{j \in N(k)} C_{kj}^y X_{kj} - B^y \right)$$

$$\text{subject to} \quad \sum_{k=1}^{K} \sum_{j \in N(k)} C_{kj}^1 X_{kj} \le B^1$$

$$\sum_{j \in N(k)} X_{kj} = 1,\ k = 1, \ldots, K \tag{4.1}$$

$$U^y \ge 0,\ y = 2, \ldots, 10$$

$$X_{kj} = 0, 1, \text{ for all } j \in N(k),\ k = 1, \ldots, K.$$

In this formulation $\{U^y\}_{y=2}^{10}$ are the nonnegative Lagrange multipliers. Since only the last nine will be relaxed, any feasible solution to this Lagrangian dual problem which satisfies the minimum performance requirement should be considered an alternative by the decision-maker, provided that the extent of deviation from the budgets for years 2-10 are acceptable.

A reduced dual problem, $Z_D(U_t)$, is formulated as the Lagrangian dual problem, Z_D, with a predetermined set of values for the vector of U^ys. The main scheme of the algorithm is to obtain an optimal solution to the Lagrangian dual problem by iteratively solving successive $Z_D(U_t)$ with a different set of values for U^ys each time. The set of Lagrangian multipliers is updated after each iteration. The algorithm involves, as the first step, solving $Z_D(U_t)$ with an initial set of values for U^ys, typically zeros. $\sum_{k=1}^{K} \sum_{j \in N(k)} R_{kj} X_{kj}$ will then be calculated and compared with the minimum performance value. If acceptable, the total costs incurred for years 2-10 will be accumulated. If the deviations from the budgets are considered as acceptable, the solution becomes a candidate solution. A check for termination will then be performed. If it fails, a new set of values for U^ys will be generated by the following rules, which constitute the standard subgradient method:

$$U^y_{t+1} = U^y_t - S_t \left(\sum_{k=1}^{K} \sum_{j \in N(k)} C^y_{kj} X^t_{kj} - B^y\right),\ y = 2, \ldots, 10 \qquad (4.2)$$

where X^t is an optimal solution to $Z_D(U_t)$ and S_t is a positive scalar step size. U_0 is defined as a predetermined initial set of values for the Lagrangian multipliers. The step size is determined by:

$$S_t = \frac{W_t(Z_D(U_t) - Z_L)}{\sum_{y=2}^{10} \left\| \sum_{k=1}^{K} \sum_{j \in N(k)} C^y_{kj} X_{kj} - B^y \right\|^2} \qquad (4.3)$$

where Z_L is a lower bound on Z_D and $0 < W_t \leq 2$. Initially, the value of Z_L can be determined, heuristically, by selecting the variable with the smallest objective coefficient from each class to represent that class. The objective value of this solution must be a lower bound on Z_D if there exists at least one feasible primal solution for the problem. Subsequently, Z_L can be improved by any feasible primal solution that has a larger objective value. $(Z_D(U_t) - Z_L)$ represents the maximum amount the incumbent optimal dual objective value can be decreased. Empirically, it seems that this type of algorithm works best with $0 < W_t < 2$ [ ]. Computational results with a variety of values for W_t are presented in a later section. Computational performance and theoretical convergence properties of the subgradient method are discussed in Held, Wolfe, and Crowder [6].

After a new set of U^ys are generated, $Z_D(U_t)$ will be solved and the whole process will be repeated until termination. An incumbent optimal value for $Z_D(U_t)$ will be kept to be compared with an incumbent optimal value for Z. The algorithm will terminate whenever these two values become "close enough". It will sometimes be the case however that this will not happen. Due to the integral duality gap, the probability that the algorithm will terminate by this criterion is highly dependent on the nature of the problem, as can be observed from the computational results. However, for all practical purposes, the algorithm will terminate upon reaching an arbitrary iteration limit, or when a predetermined number of acceptable candidate solutions are generated. Due to the adjustment of the different parameters, proof of convergence is not possible. A branch-and-bound scheme is incorporated into the algorithm such as to theoretically assure convergence.

By a close examination of the reduced dual problem, $Z_D(U_t)$, it is obvious that this subproblem takes on the form of a multiple choice knapsack problem, except possibly with negative objective coefficients. The authors utilized the method by Sinha and Zoltners for the multiple choice knapsack problem [8] here in solving this reduced dual problem.

The choice is based on the quickness and ease of reoptimization of the method, and also its ability to blend in with the branch-and-bound scheme without much additional effort.

The branch-and-bound scheme involved is the partitioning of the set of variables in any multiple choice class into two subsets. On one branch of the tree, a subset of the variables will be set to zero, while on the other branch, the rest of the variables will be set to zero. The LIFO method is chosen in branching and backtracking due to the ease of reoptimization and the minimal storage requirement.

The following is a step-by-step description of the algorithm:

Step 1: Let $Z_D^* = \infty$, $Z^* = -\infty$, $t = 0$. Initialize U_t.

Step 2: Solve $Z_D(U_t)$. If $Z_D(U_t) < Z_D^*$, $Z_D^* = Z_D(U_t)$.

Step 3: Check for feasibility of Z with X_t.
If feasible and $Z > Z^*$, $Z^* = Z$.

Step 4: If $Z^* = Z_D^*$, STOP.
Otherwise, continue.

Step 5: Let $t = t + 1$. Update U_t by (4.2) and (4.3).
Go to step 2.

With Lagrangian relaxation in integer programming, a duality gap almost always exists. The difficulty of proving optimality can be resolved by settling for solutions which lie within a predetermined percentage (say within 1%) of optimality. That is, if Z^* becomes greater than or equal to 99% of Z_D^*, the procedure will be terminated. Since only a standardized branch-and-bound method is utilized in the algorithm, it will not be discussed here. Basically, it involves first selecting an initial set of U_t. Then $Z_D(U_t)$ will be solved a fixed number of times by modifying U_t using (4.2) and (4.3). If the procedure fails to stop, the branching scheme will take place. Since the LIFO strategy is being applied, it is more efficient, for re-optimization purposes, to use the last set of values for U_t to be the initial values for the succeeding branch, instead of re-setting U_t to zeros. Generally, upon termination of the algorithm. a set of alternative solutions will have been generated. The user can then select the most appropriate solution, taking into considerations factors that cannot be captured by the mathematical model.

5. IMPLEMENTATION AND COMPUTATIONAL RESULTS

Due to a lack of a theoretical scheme for evaluating the performance of this type of problem, the authors have coded the above algorithm in standard FORTRAN. Test problems were then generated so as to observe the behavior of the algorithm, empirically, based on the variations of different parameters. All tests were performed on the IBM 370 system at York University. Data from actual applications of the model were used in the tests. Problems of the following sizes were included: 50 objects, 95 projects and 212 projects. The authors feel that randomly generated problems will not be able to provide significant insight here due to the special requirements of the model. A close examination of the data reveals the fact that in this type of problem, a typical variable (or alternative) will have one or two very large constraint coefficients relative to the others, for the ten constraints.

This creates a very unstable system when one switches from one alternative to another. Additionally within the same project, the coefficients for the different alternatives have practically the same values, because they are mainly maintenance costs, one-lift costs, or two-lift costs. Out of a project with one hundred alternatives, there might only be five or six values which differ significantly from one another for the first year budget constraint. This renders the reduced dual problem, $Z_D(U_t)$, very insensitive.

As indicated by the test results, the solution procedure for the problem becomes more stable as the number of projects increases. The W parameters perform best with values in the neighbourhood of 0.5, due to the similarity of the coefficients. One significant observation is that the values of the Lagrangian multipliers tend to fluctuate, up and down, drastically. A remedy that works well, empirically, is to decrease the values of the multipliers at a rate that is half of the rate of increase. In other words, during the updating of the U^ys, those that will increase in value will do so at a certain rate, while those that need to be decreased will take on a rate half of the rate of increase. Also it is common practice in gradient method to reduce the step-size, W, by half whenever Z_D fails to improve in a fixed number of steps.

The storage requirement for this algorithm is minimal. With N equal to the total number of variables and K the number of road sections (or projects), the major storage requirement is approximately 16N+21K, including storage for the original coefficients. Storage for the branch-and-bound method is additional, it is a function that depends on the depth of the tree. From empirical results, this storage is negligible.

The solution time is relatively fast for the sample problems tested. For example, a typical problem with 200 road sections and 100 alternatives in each section can be solved in approximately 400 CPU seconds (350 $Z_D(U_t)$ solved) on an IMB 370. Additional results are printed in Table 1.

It has been found, at least with these sample data, that with this type of problem it is relatively easy to locate candidate solutions, while 99% optimal solution may be difficult to find. For the 212 project case it was not possible in one case to locate such a solution within the iteration limit. But, overall, the solution times were satisfactory, and the candidate solutions generated were adequate.

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Table 1

Computational Results

(Approximately 100 alternatives for each project)
(Execution time is in CPU seconds on IBM 370)
(Problems are solved until within 95% optimal)

212 Projects

	W = 1.0	W = 1.0/0.5	W = 0.5/0.25
Number of subproblems solved	2015*	502	210
Execution time (seconds)	2086	524	216
Number of primal feasible solutions generated	80	109	45

93 Projects

	W = 1.0	W = 1.0/0.5	W = 0.5/0.25
Number of subproblems solved	1087	240	121
Execution time (seconds)	259	59	32
Number of primal feasible solutions generated	34	11	9

* REACHED ITERATION COUNT LIMIT.

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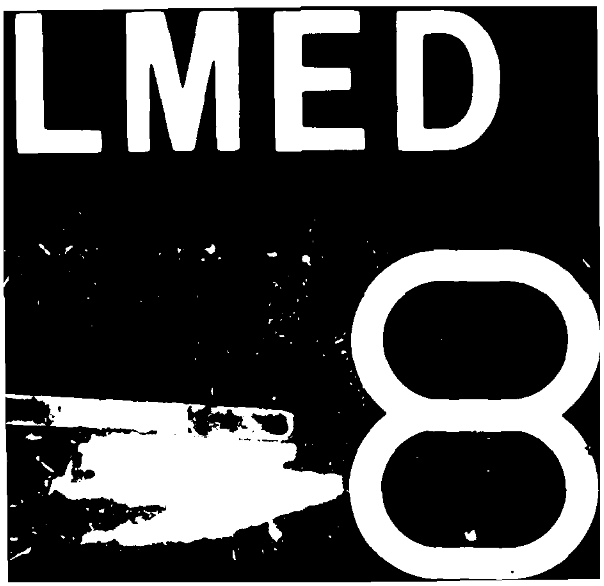
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